Q1

1

Fully expand $(4-x)^4$.

$a=4 \quad b=-x \quad n=4$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

$$(4-x)^4 = 4^4 + {}^4C_1 4^3(-x) + {}^4C_2 4^2 (-x)^2 + {}^4C_3 4^1 (-x)^3 + 4^0(-x)^4$$

$$= \boxed{256 - 256x + 96x^2 - 16x^3 + x^4}$$

Q2

2

Fully expand $(2-\frac{1}{3}x)^4$.

$a=2 \quad b=\frac{-x}{3} \quad n=4$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

$$\left(2-\frac{x}{3}\right)^4 = 2^4 + {}^4C_1 2^3\left(-\frac{x}{3}\right) + {}^4C_2 2^2\left(-\frac{x}{3}\right)^2 + {}^4C_3 2^1\left(-\frac{x}{3}\right)^3 + \left(-\frac{x}{3}\right)^4$$

$$= 16 + 4(8)\left(-\frac{x}{3}\right) + 6(4)\left(\frac{x^2}{9}\right) + 4(2)\left(-\frac{x^3}{27}\right) + \frac{x^4}{81}$$

$$= \boxed{16 - \frac{32x}{3} + \frac{8x^2}{3} - \frac{8x^3}{27} + \frac{x^4}{81}}$$

Q3

3

Find the coefficient of the term in x^4 in the expansion of $(3+2x)^9$.

$a=3 \quad b=2x \quad n=9$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

when $r=4$

${}^9C_4 3^5 (2x)^4$

$= 126(243)16x^4$

$= \underbrace{489888}x^4$

$\boxed{\text{coefficient} = 489888}$

Q4a

4a

(a) Find the first three terms, in ascending powers of x, in the expansion of $(5-2x)^4$.

$a=5 \quad b=-2x \quad n=4$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

$(5-2x)^4 \approx 5^4 + {}^4C_1 5^3(-2x) + {}^4C_2 5^2(-2x)^2$

$\approx \boxed{625 - 1000x + 600x^2}$

Q4b

4b

(b) Use your answer to part (a) to estimate $(4.5)^4$.

Answer from (a): $(5-2x)^4 \approx 625 - 1000x + 600x^2$

Let $4.5 = 5 - 2x$

$x = 0.25$

Sub $x = 0.25$ into approximation in (a)

$(4.5)^4 \approx 625 - 1000(0.25) + 600(0.25)^2$

$\approx \boxed{412.5}$

Q5

5

In the expansion of $(4 - px)^6$, the coefficient of the x^4 term is 19 440.
Given that p is a positive integer find the value of p.

coefficient of $x^4 = 19440$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$

$a = 4 \quad b = -px \quad n = 6$

When $r = 4$

${}^6C_4\, 4^2 (-px)^4$

$240p^4 x^4$

$\xrightarrow{}$ coefficient $= 240p^4 = 19440$

$p = \sqrt[4]{\frac{19440}{240}}$

$= \sqrt[4]{81}$

$= \pm 3$

p must be a +ve integer $\therefore$ $\boxed{p = 3}$

Q6

6

In the expansion of $(3a - 2x)^6$, the coefficient of the x^3 term is equal to the coefficient of the x^4 term. Find the value of a.

$a \Rightarrow 3a \quad b \Rightarrow -2x \quad n = 6$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

When $r = 3$

${}^6C_3 (3a)^{6-3} (-2x)^3$

$\underbrace{20 (27a^3)(-8x^3)}$

When $r = 4$

${}^6C_4 (3a)^{6-4} (-2x)^4$

$\underbrace{15 (9a^2)(16x^4)}$

equate coefficients $-4320a^3 = 2160a^2$

$\boxed{a = -0.5}$

Q7a

7a

(a) Find the first three terms in the expansion of $(2 - 3x)^7$.

(b) Given that x is small such that x^3 and higher powers of x can be ignored show that
$$(1 - 2x)(2 - 3x)^7 \approx 128 - 1600x + 8736x^2$$

a) $a = 2 \quad b = -3x \quad n = 7$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

$(2-3x)^7 \approx {}^7C_0 2^7 (-3x)^0$

$+ {}^7C_1 2^6 (-3x)$

$+ {}^7C_2 2^5 (-3x)^2$

$\approx \boxed{128 - 1344x + 6048x^2}$

Q7b

7b

(a) Find the first three terms in the expansion of $(2-3x)^7$.

$(2-3x)^7 \approx 128 - 1344x + 6048x^2$

(b) Given that x is small such that x^3 and higher powers of x can be ignored show that
$(1-2x)(2-3x)^7 \approx 128 - 1600x + 8736x^2$

$(1-2x)(2-3x)^7 \approx (1-2x)(128 - 1344x + 6048x^2)$

$\approx 128 - 1344x + 6048x^2$

$-256x + 2688x^2 - \cancel{12096x^3}$ ignore

$\approx \boxed{128 - 1600x + 8736x^2}$

Q8

8

In the expansion of $(p+qx)^8$, the coefficients of the x^2 term and the x^6 term are equal.
Find p in terms of q.

[3]

$a = p \quad b = qx \quad n = 8$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

in x^2 term $r = 2$

${}^8C_2 p^6 (qx)^2$

$28p^6q^2x^2$

in x^6 term $r = 6$

${}^8C_6 p^2 (qx)^6$

$28p^2q^6x^6$

$\cancel{28}p^{\cancel{6}\,4}\cancel{q^2} = \cancel{28}\cancel{p^2}q^{\cancel{6}\,4}$

$p^4 = q^4$

$\boxed{p = \pm q}$

Q9

9

In the expansion of $(1+x)^n$, the coefficient of the x^3 term is 84.
Find the value of n.

$a = 1 \quad b = x \quad n = n$

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$

$r = 3$

${}^nC_r = \dfrac{n!}{r!\,(n-r)!}$

${}^nC_3\, 1^{n-3} x^3$

$= \dfrac{n!}{3!\,(n-3)!} x^3$

$3! = 3 \times 2 \times 1 = 6$

$n! = n(n-1)(n-2)(n-3)\ldots 1$

$= \dfrac{n(n-1)(n-2)\cancel{(n-3)\ldots 1}}{6\,\cancel{(n-3)(n-4)(n-5)\ldots 1}} x^3$

$= \dfrac{n(n-1)(n-2)}{6} x^3$

coefficient: $\dfrac{n(n-1)(n-2)}{6} = 84$

$n(n-1)(n-2) = 504$

What 3 consecutive numbers multiply to get 504?

If $\sqrt[3]{504} = 7.96$ (3sf)

Lets try 7, 8, 9 $\quad 7 \times 8 \times 9 = 504$

$\boxed{n = 9}$

Q10

10

In the expansion of $(a+bx)^4$, the coefficient of the x^3 term is 216. ①
In the expansion of $(a+bx)^6$, the coefficient of the x^4 term is 4860. ②
Find the possible values of a and b.

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$

①	②
$a = a \quad b = bx \quad n = 4$	$a = a \quad b = bx \quad n = 6$
$r = 3$	$r = 4$
${}^4C_3\, a^1 (bx)^3$ $\underbrace{4ab^3}x^3$	${}^6C_4\, a^2 (bx)^4$ $\underbrace{15a^2b^4}x^4$

$4ab^3 = 216 \qquad 15a^2b^4 = 4860$

③ $ab^3 = 54 \qquad a^2b^4 = 324$ ④

solve simultaneous equns to find a, b

$a = \dfrac{54}{b^3}$ sub into eqn ④

$\left(\dfrac{54}{b^3}\right)^2 b^4 = 324$

$9 = b^2$

$b = \pm 3$

$a = \dfrac{54}{(\pm 3)^3} = \pm 2$

$\boxed{a = \pm 2,\ b = \pm 3 \text{ either both +ve or both -ve}}$